
CONSECUTIVE INTEGERS – LINEAR

❑ INTRODUCTION

Consecutive Integers

An **integer** is either a positive whole number, zero, or a negative whole number; in other words, it's the collection of numbers:

$\dots -4, -3, -2, -1, 0, 1, 2, 3, 4 \dots$ The set of **integers**

An example of four **consecutive integers** might be **11, 12, 13, 14**. Notice that consecutive integers differ by 1 — each integer is 1 more than the previous integer (assuming they're listed in ascending order). Better yet, we can think of the 12, the 13, and the 14 in the following way:

Start with **11** (the smallest of the 4 numbers)

$$11 + \underline{1} = \mathbf{12}$$

$$11 + \underline{2} = \mathbf{13}$$

$$11 + \underline{3} = \mathbf{14}$$



This means that if we have four unknown consecutive integers, and if we call the smallest one x , then the next one would be written $x + 1$, and the one after that, $x + 2$, and the largest one, $x + 3$.

$$x$$

$$x + 1$$

$$x + 2$$

$$x + 3$$

Four **consecutive integers**

Consecutive *Even* Integers

An example of three **consecutive even integers** might be **8, 10, 12**. Notice that consecutive even integers differ by 2 and that we usually write them in ascending order (smallest to largest).

Thus, in a sequence of consecutive even integers, each number after the first number is 2 larger than the preceding number. So, assuming the smallest of them is called x , they could be written

$$\begin{array}{l} x \\ x + 2 \\ x + 4 \end{array} \quad \text{Three consecutive even integers}$$

Consecutive *Odd* Integers

An example of five **consecutive odd integers** might be $-11, -9, -7, -5, -3$. Notice that consecutive odd integers differ by 2. Now comes a curious little fact: If n is an odd integer, then the odd integer that comes right after n must be $n + 2$, since they must differ by 2. (For example, -11 is odd, and the next odd integer is $-11 + 2 = -9$. ✓) Therefore, if x represents the smallest of five consecutive odd integers, for instance, they could be denoted like this:

$$\begin{array}{l} x \\ x + 2 \\ x + 4 \\ x + 6 \\ x + 8 \end{array} \quad \text{Five consecutive odd integers}$$

□ **EXAMPLES**

EXAMPLE 1: Four consecutive odd integers have a sum of 32. Find the four numbers.

Solution: If we let n represent the smallest of the four consecutive odd integers, then the four integers can be represented like this: $n, n + 2, n + 4$, and $n + 6$. Since the given condition is that their sum must be 32, the following equation seems appropriate:

$$n + (n + 2) + (n + 4) + (n + 6) = 32$$

Dropping the parentheses (which weren't really necessary):

$$4n + 12 = 32 \Rightarrow 4n = 20 \Rightarrow n = 5$$

Since n , the smallest of the four numbers, is 5, our final answer is

5, 7, 9, 11

EXAMPLE 2: Find three consecutive integers such that the largest is 18 less than 3 times the smallest.

Solution: We can represent the three consecutive integers with a , $a + 1$, and $a + 2$. Is it clear that a is the smallest and $a + 2$ is the largest? The relationship between the largest and the smallest, as stated in the problem, can be transformed into the following equation:

$$a + 2 = 3a - 18$$

$$\Rightarrow -2a = -20$$

$$\Rightarrow a = 10$$

Since the smallest of the three consecutive integers is 10, the three numbers must be

10, 11, 12

EXAMPLE 3: If the mean (average) of five consecutive even integers is 2, find the largest of the integers.

Solution: We can represent the five consecutive even integers with x , $x + 2$, $x + 4$, $x + 6$, and $x + 8$. Recall that the mean of five numbers is found by dividing the sum of the numbers by 5:

$$\frac{x + (x + 2) + (x + 4) + (x + 6) + (x + 8)}{5} = 2$$

$$\begin{aligned} \Rightarrow \frac{5x + 20}{5} &= 2 && \text{(combine like terms)} \\ \Rightarrow 5x + 20 &= 10 && \text{(multiply each side by 5)} \\ \Rightarrow 5x &= -10 && \text{(subtract 20 from each side)} \\ \Rightarrow x &= -2 && \text{(divide each side by 5)} \end{aligned}$$

Since $x = -2$, and the largest of the five integers is $x + 8$, it follows that the largest integer is $-2 + 8 = \boxed{-6}$

Homework

1. Find three consecutive integers whose sum is 57.
2. Find four consecutive even integers whose sum is 52.
3. Find three consecutive odd integers whose sum is 39.
4. Find three consecutive integers such that the sum of the smallest and largest is 100.
5. Find five consecutive even integers such that the largest is 3 times the smallest.
6. Find four consecutive odd integers such that the largest is 3 less than twice the smallest.
7. Find three consecutive integers such that the largest number is twice the middle number.
8. Find four consecutive even integers such that twice the smallest plus 4 times the largest equals 0.

9. Find three consecutive even integers such that the sum of the smallest and the largest is 20.
10. Find five consecutive odd integers such that the largest is 4 less than 5 times the smallest.
11. The mean (average) of four consecutive integers is 8.5. Find the 4 numbers. Be sure you write and solve an equation.
12. Find three consecutive odd integers whose mean is 21.
13. Find five consecutive even integers whose mean is 0.

Solutions

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|----------------------------|---------------------------|-------------------------|
| 1. 18, 19, 20 | 2. 10, 12, 14, 16 | 3. 11, 13, 15 |
| 4. 49, 50, 51 | 5. 4, 6, 8, 10, 12 | 6. 9, 11, 13, 15 |
| 7. 0, 1, 2 | 8. -4, -2, 0, 2 | 9. 8, 10, 12 |
| 10. 3, 5, 7, 9, 11 | 11. 7, 8, 9, 10 | 12. 19, 21, 23 |
| 13. -4, -2, 0, 2, 4 | | |

**“EDUCATION IS THE
PASSPORT TO THE FUTURE,
FOR TOMORROW
BELONGS TO THOSE
WHO PREPARE FOR
IT TODAY.”**



Malcolm X (1925–1965)